Understanding Electromagnetic Form Factors

Gerald A. Miller
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What do form factors really measure?

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What do form factors really measure?

Not the 3 Dimensional Fourier transform of the charge or magnetization density

Outline

1. Introduction,motivation

2. Why the charge density is not the 3-dimensional Fourier transform of G_E, Toy model

3. Use of light front/infinite momentum frame

4. Model independent neutron transverse charge density GAM PRL 99:112001,2007

5. Basic considerations for neutral systems, toy model

6. Exclusive-inclusive connection
elastic- deep-inelastic
John Arrington, GAM PRC78, 032201(R) 2008

7. Relation between transverse density and 3-dimensional density

Experimental progress

Why form factors?

- UNDERSTAND CONFINEMENT
- How does the nucleon stick together when struck by photon?
- Where is charge and magnetization density located?
- Origin of angular momentum?
- What is the shape of the proton?

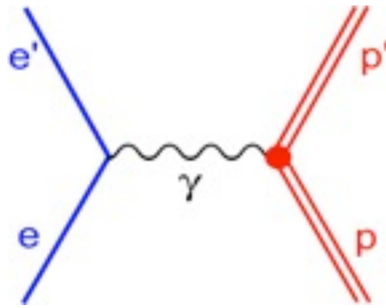
What is charge density at the center of the neutron?

- **Neutron has no charge, but charge density need not vanish**
- **Is central density positive or negative?**

Fermi: n fluctuates to $p\pi^-$ **p at center, pion floats to edge**

Electron scattering from a nucleon

$j_\mu = \langle e' | \gamma_\mu | e \rangle$



$J_\mu = \langle p' | \Gamma_\mu | p \rangle$

Nucleon vertex:

$$\Gamma_\mu(p,p') = \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M} F_2(Q^2)$$

Dirac **Pauli**

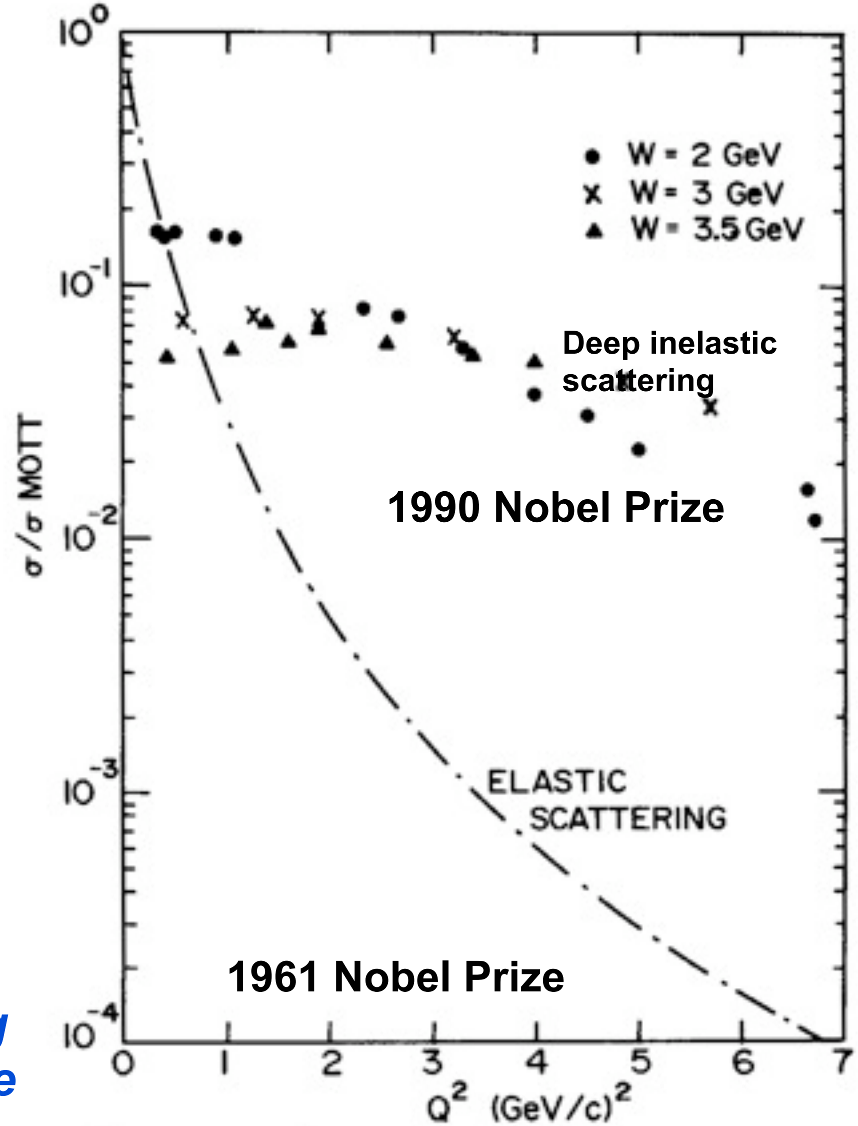
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(G_E^2 + \frac{\tau}{\epsilon} G_M^2\right) / (1+\tau)$$

Cross section for scattering from a point-like object

G_E, G_M Sachs

Form factors describing nucleon shape/structure

$$G_E(Q^2) \equiv F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2),\ G_M(Q^2) \equiv F_1(Q^2) + F_2(Q^2)$$



Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

Correct non-relativistic:

wave function invariant under Galilean transformation

Relativistic : wave function is frame dependent, initial and final states differ

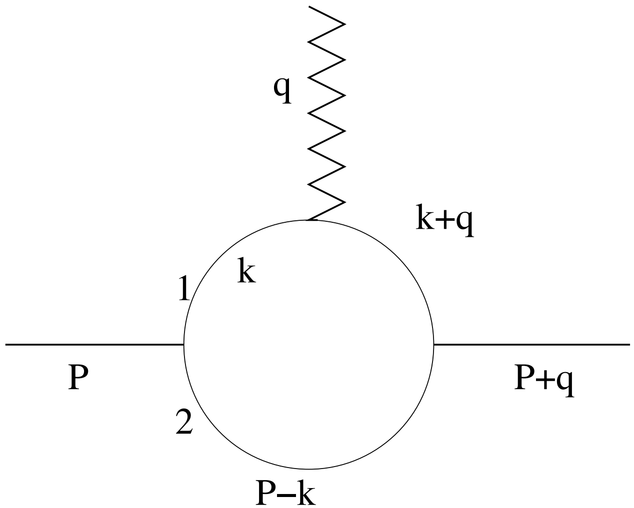
interpretation of Sachs FF is wrong

Final wave function is boosted from initial

Need relativistic treatment

Toy model

- Scalar meson, mass M made of two scalars one neutral, one charged of mass m, with M<2m (stable particle)
- Exact covariant calculation of form factor



Infinite momentum frame (Light front variables) gives exact result

When non-relativistic approximation works, Form factor is 3DFT of charge density.

When does non-relativistic approximation work?

Validity of non-relativistic approximation:

M=2m-B, B=0.002 M, $Q^2 \leq 0.2M^2$

very limited

only deuteron kinematics are non-rel

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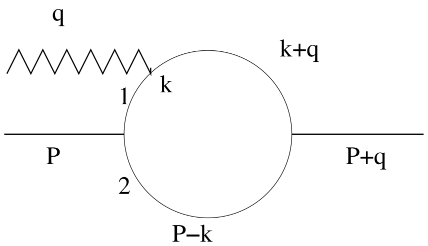
only deuteron kinematics are non-rel

Relativity needed

Rest frame charge density is not observable

(a)



(b)

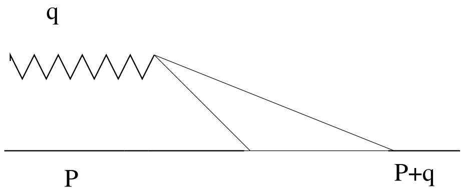


Fig. a. $\langle P+q|J^\mu(0)|P\rangle = (2P+q)^\mu F(Q^2) \to \frac{g^2}{(2\pi)^3}\int \frac{d^3p}{2E_1{E_1}'2E_2}\frac{(p_1^\mu+{p'}_1^\mu)}{(E_P-E_1-E_2)(E_{\mathbf{P+q}}-{E'}_1-E_2)}$ correct in IMF ($P\to\infty$).

Target rest frame: $(p_1^\mu + {p'}_1^\mu) = [E_1 + {E'}_1, 2\mathbf{p}+\mathbf{q}]$.

$$I_1(\mathbf{q}^2) \equiv \int \frac{d^3p}{2E_1{E_1}'2E_2}\frac{(\sqrt{p^2+m^2}+\sqrt{(\mathbf{p}+\mathbf{q})^2+m^2})}{(E_P-E_1-E_2)(E_{\mathbf{P+q}}-{E'}_1-E_2)}$$
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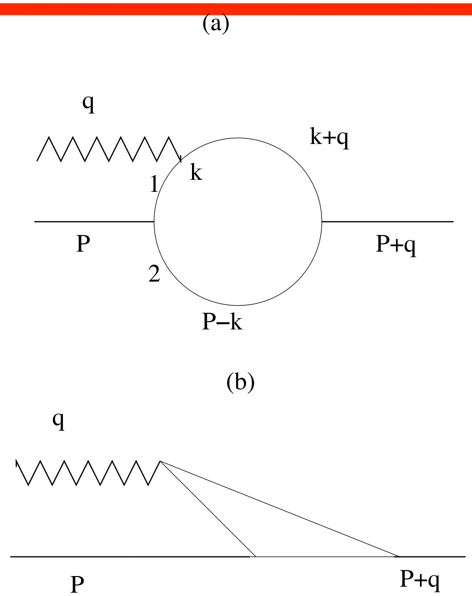
$$\hat{\mathbf{q}}J_3(\mathbf{q}^2) \equiv \int \frac{d^3p}{2E_1{E_1}'2E_2}\frac{\mathbf{q}}{(E_P-E_1-E_2)(E_{\mathbf{P+q}}-{E'}_1-E_2)}.$$

$$(2P+q)^\mu F(Q^2) \to \frac{g^2}{(2\pi)^3}[I_1, \hat{\mathbf{q}}(J_2+J_3)], \quad q_\mu J^\mu = 0? = q^0 I_1 - |\mathbf{q}|(J_2+J_3) \equiv CC,$$

Rest frame charge density is not observable



Curr. Cons. massively violated

Fig. a. $\langle P+q|J^\mu(0)|P\rangle = (2P+q)^\mu F(Q^2) \to \frac{g^2}{(2\pi)^3}\int \frac{d^3p}{2E_1{E_1}'2E_2}\frac{(p_1^\mu+{p'}_1^\mu)}{(E_P-E_1-E_2)(E_{\mathbf{P}+\mathbf{q}}-{E'}_1-E_2)}$ correct in IMF ($P\to\infty$).

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Light cone coordinates/Infinite momentum frame

$$\text{“Time”}\ x^+ = (ct+z)/\sqrt{2} = (x^0+x^3)/\sqrt{2}$$

$$\text{“Evolution”}\quad p^- = (p^0-p^3)/\sqrt{2}$$

$$\text{“Space”}\ x^- = (ct-z)/\sqrt{2} = (x^0-x^3)/\sqrt{2},\ \text{If}\ x^+ = 0,\ \ x^- = -\sqrt{2}z$$

$$\text{“Momentum”}\quad p^+ = (p^0+p^3)/\sqrt{2}$$

Transverse : “Position” b “Momentum” p **perp**

Relativistic formalism- kinematic subgroup of Poincare

- Lorentz transformation –transverse velocity v

$$k^+ \to k^+, \; \mathbf{k} \to \mathbf{k} - k^+\mathbf{v}$$

k^- such that k^2 not changed

Just like non-relativistic with k^+ as mass, take momentum transfer in perp direction, then density is 2 Dimensional Fourier Transform

Infinite Momentum Frame Charge Density

$$\widehat{\rho}_\infty(x^-,\mathbf{b}) = \sum_q e_q \bar{q}(x^-,\mathbf{b})\gamma^+ q(x^-,\mathbf{b}) \quad = J^+(x^-,\mathbf{b})$$

$$|p^+,\mathbf{R}=\mathbf{0},\lambda\rangle \equiv \mathcal{N}\int \frac{d^2\mathbf{p}}{(2\pi)^2}|p^+,\mathbf{p},\lambda\rangle$$

$$\rho_\infty(x^-,\mathbf{b}) = \langle p^+,\mathbf{R}=\mathbf{0},\lambda|\,\hat{\rho}_\infty(x^-,\mathbf{b})\,|p^+,\mathbf{R}=\mathbf{0},\lambda\rangle$$

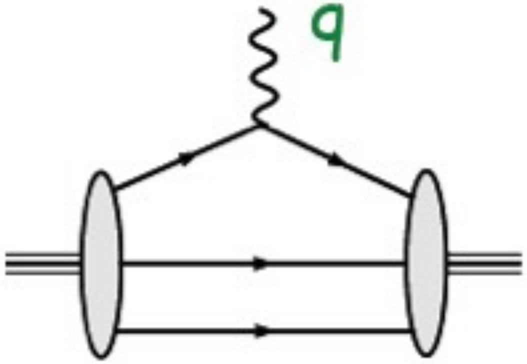
Integrate over x^-, use momentum expansion, definition of F_1:

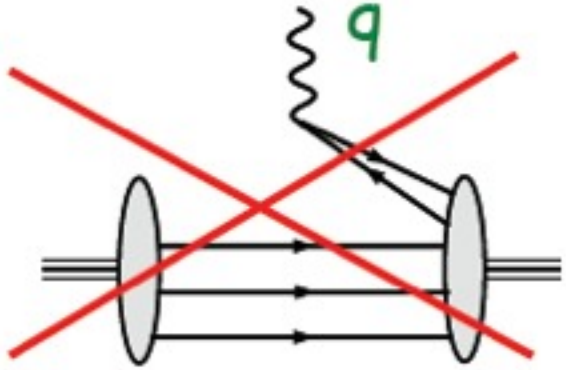
$$\boxed{\rho(b) \equiv \int dx^- \rho_\infty(x^-,\mathbf{b}) = \int \frac{d^2q}{(2\pi)^2} F_1(Q^2=\mathbf{q}^2)e^{-i\mathbf{q}\cdot\mathbf{b}}}$$

Transverse charge density

$$F_1 = \langle p^+,\mathbf{p}',\lambda|J^+(0)|p^+,\mathbf{p},\lambda\rangle$$

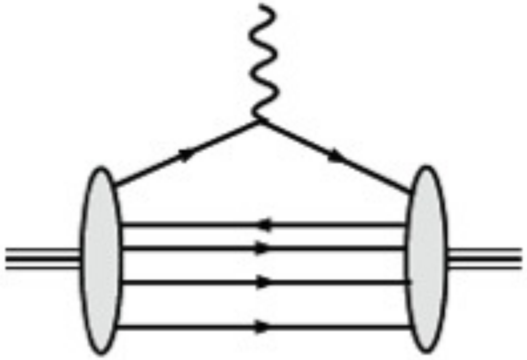
interpretation of FF as quark density





overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation



overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density

Absent in a Drell-Yan Frame

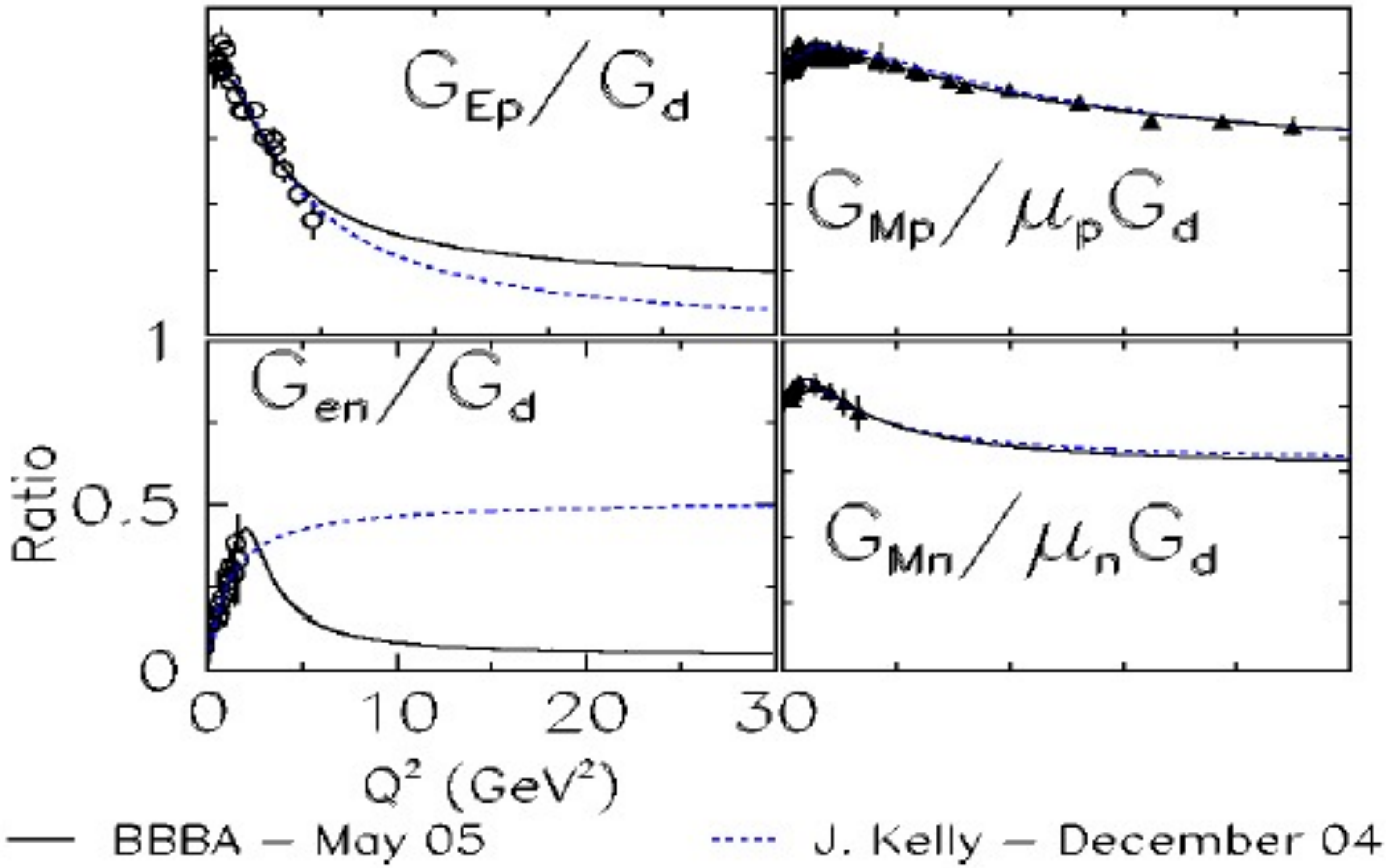
$$\boxed{q^+ = q^0 + q^3 = 0}$$

From Marc Vanderhaeghen

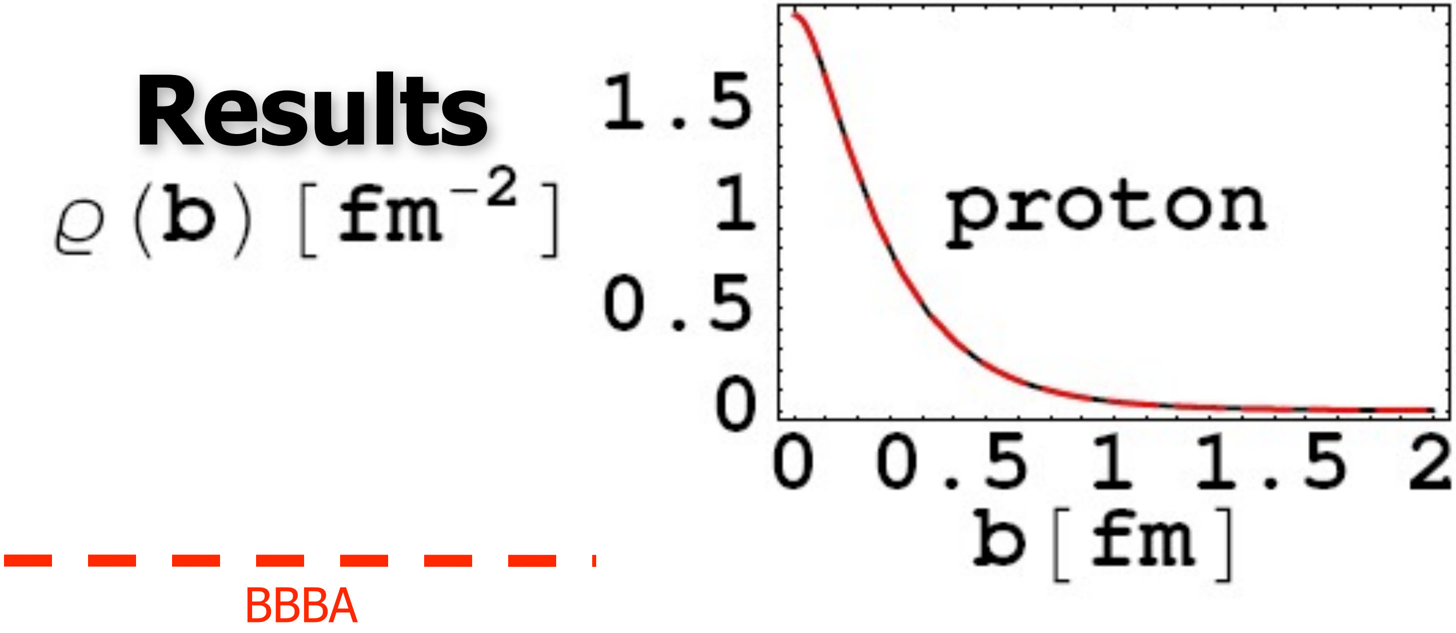
Parameterizations of form factors-new data not in

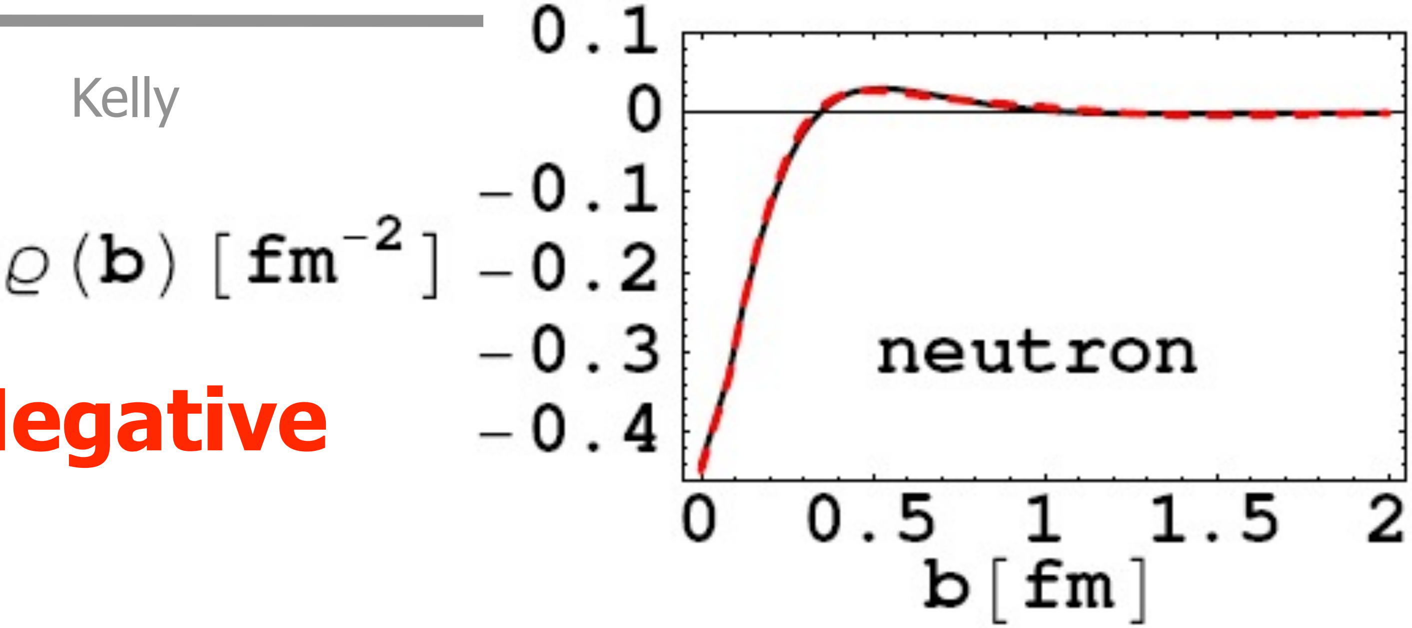
R. Bradford,[a] A. Bodek,[a] H. Budd,[a] and J. Arrington[b]

hep-ex/0602017



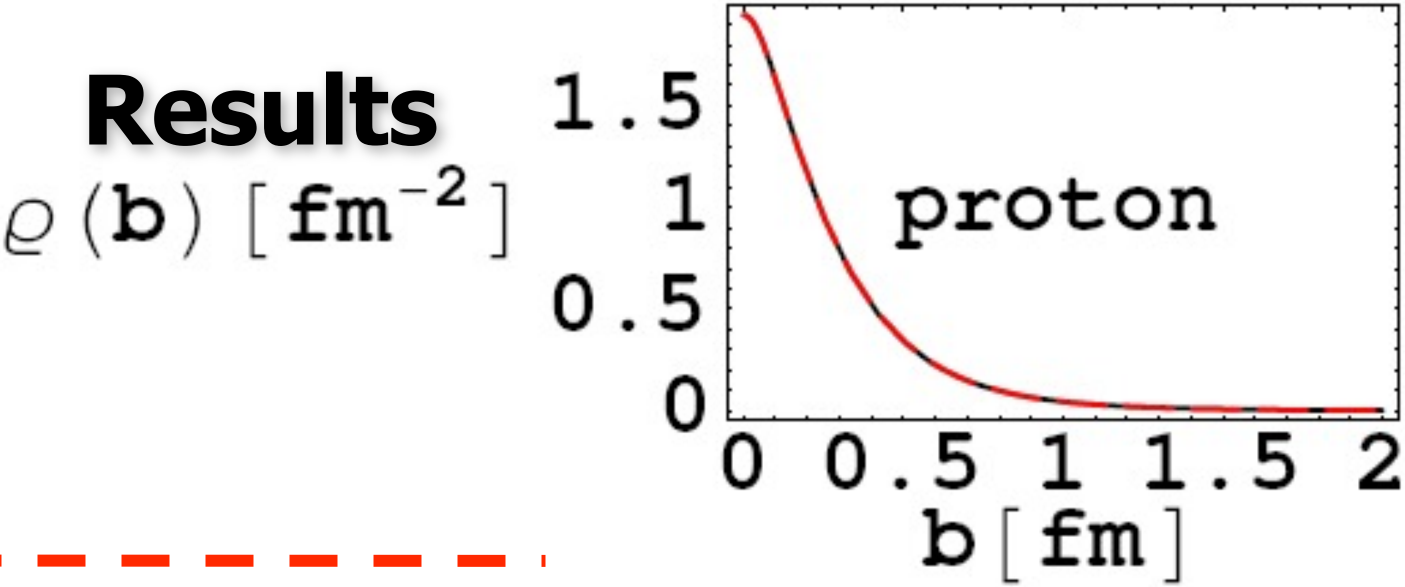
Results





Negative

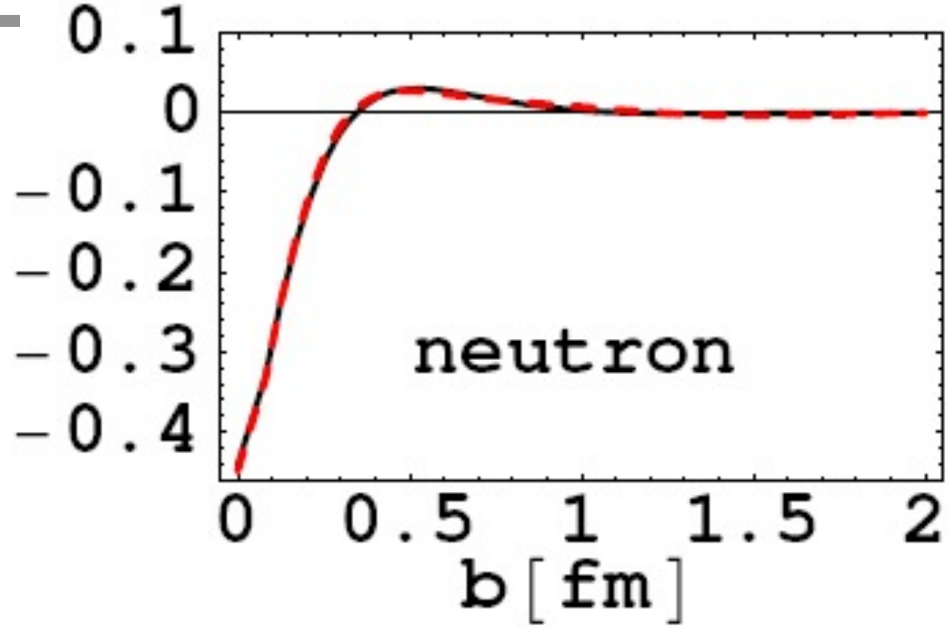
Results



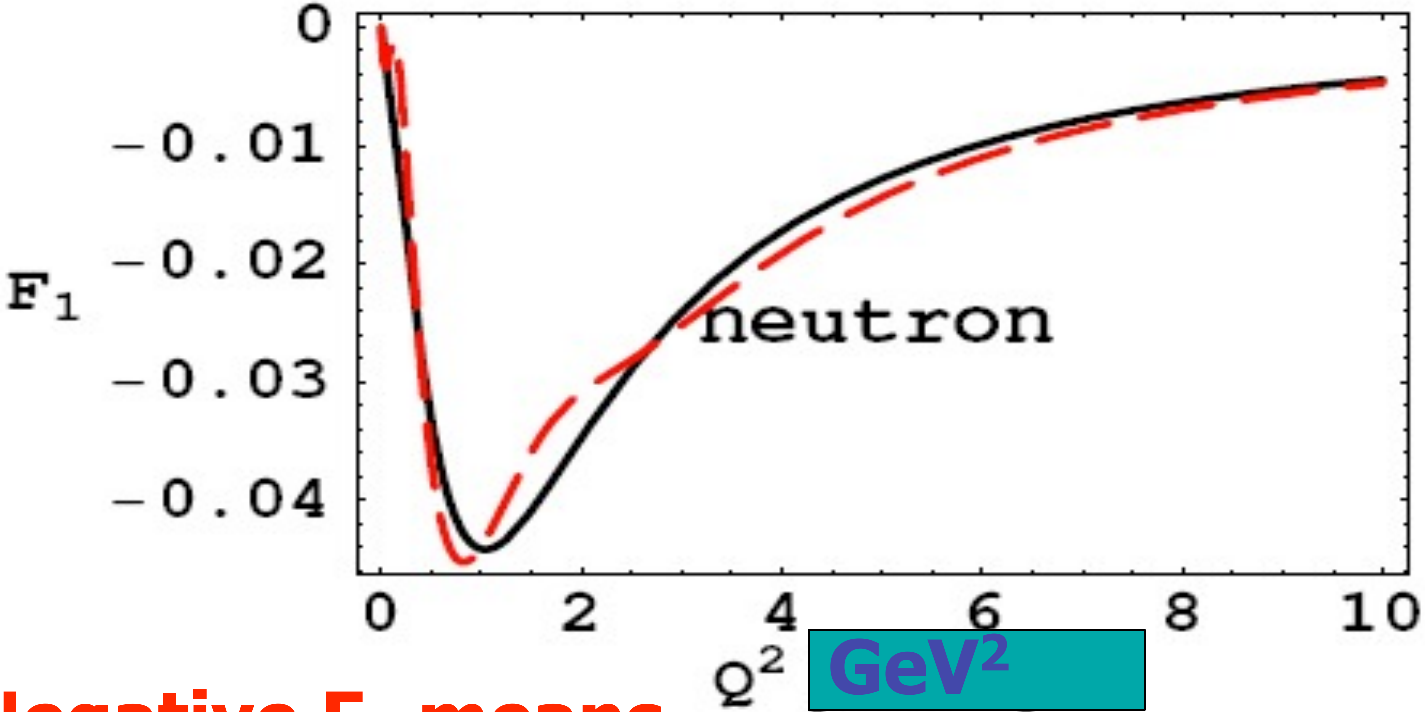
BBBA

Kelly

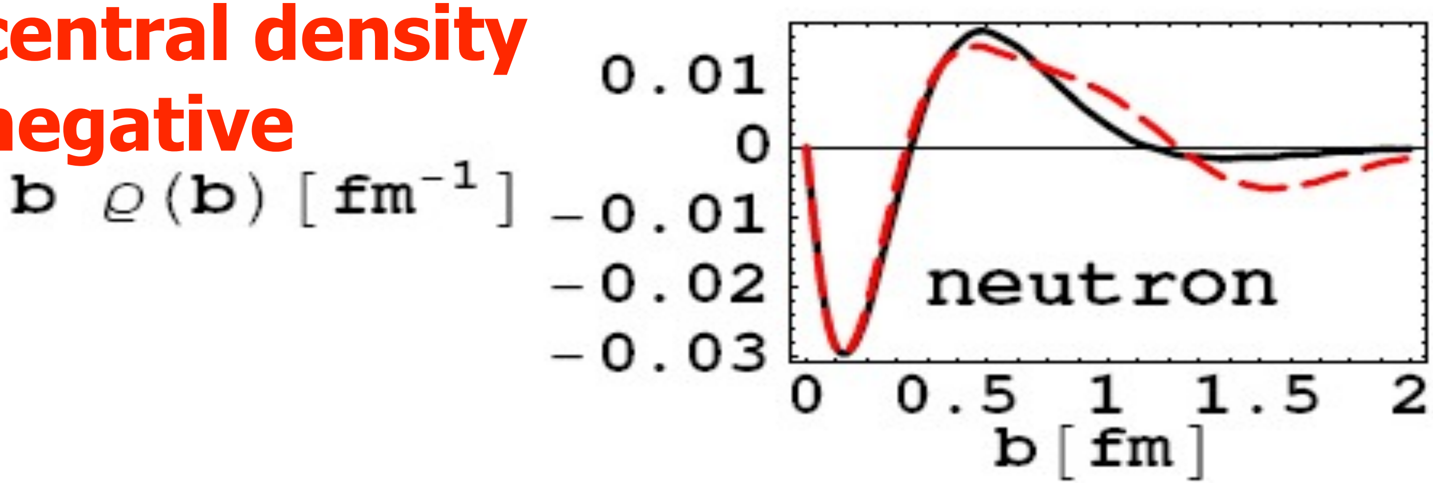
$\rho(\mathbf{b})\ [\mathrm{fm}^{-2}]$



Negative

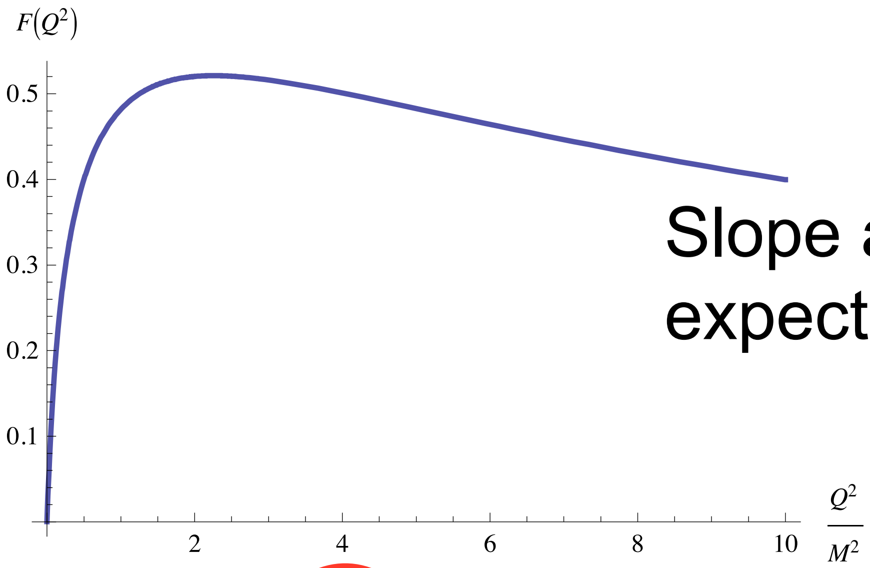


Negative F_1 means central density negative



Neutral systems basic intuition

- particle 1, + charge, $m_1 = M$
- particle 2, - charge, $m_2 = 0.14 m_1$

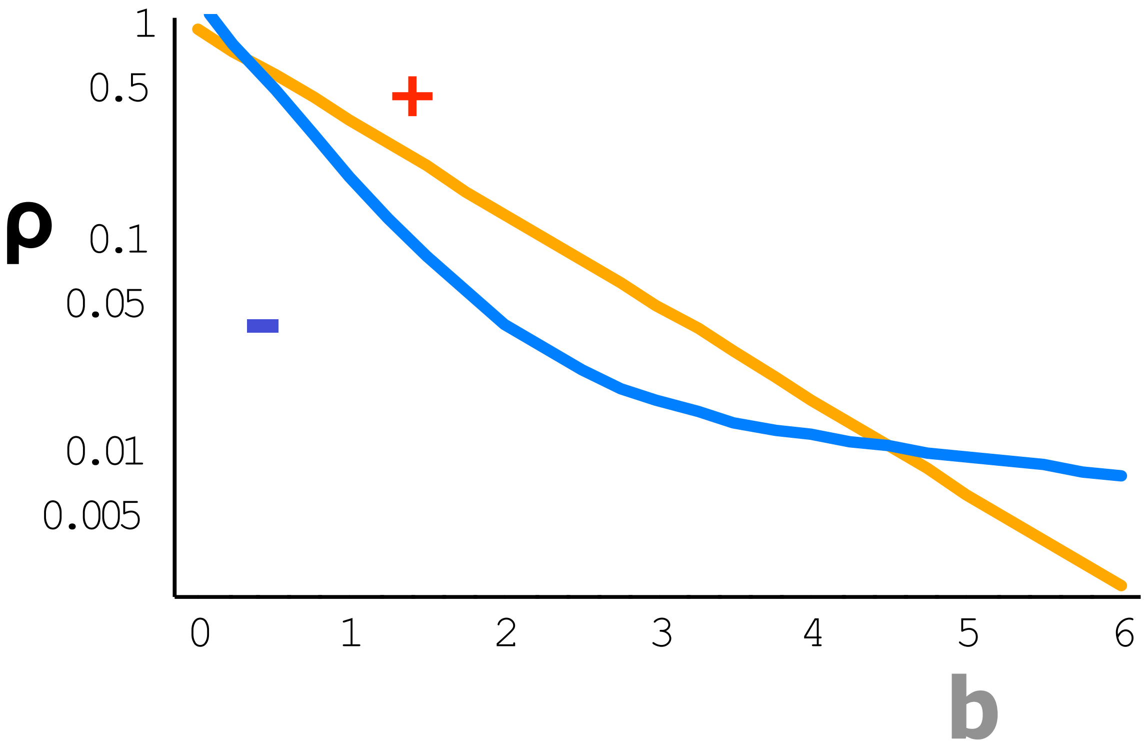


Slope at origin is positive, as expected

$$M^2 R^2 = -\frac{g^2}{4\pi^2}\left(\frac{M^2}{48 m_2^2} + \frac{1}{9}\left(1 - 4\log\left(\frac{m_2^2}{M^2}\right)\right) + \frac{11}{512}\pi\sqrt{\frac{m_2^2}{M^2}} - \frac{5\pi}{192\sqrt{\frac{m_2^2}{M^2}}} - \frac{7 m_2^2}{288 M^2}\right)$$

Why is neutron different

Neutron Interpretation needed



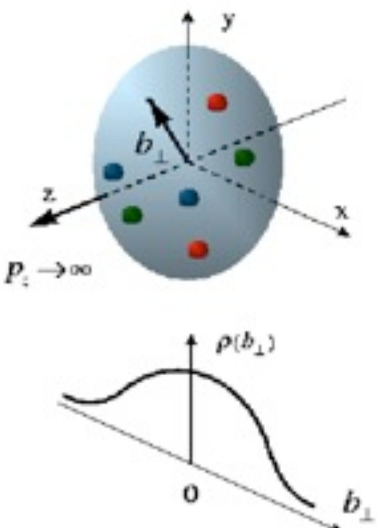
Why ? What? How? Combine elastic and deep inelastic scattering information.

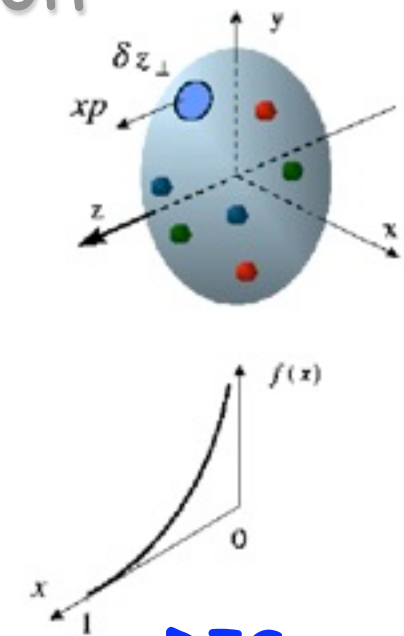
Generalized parton distribution

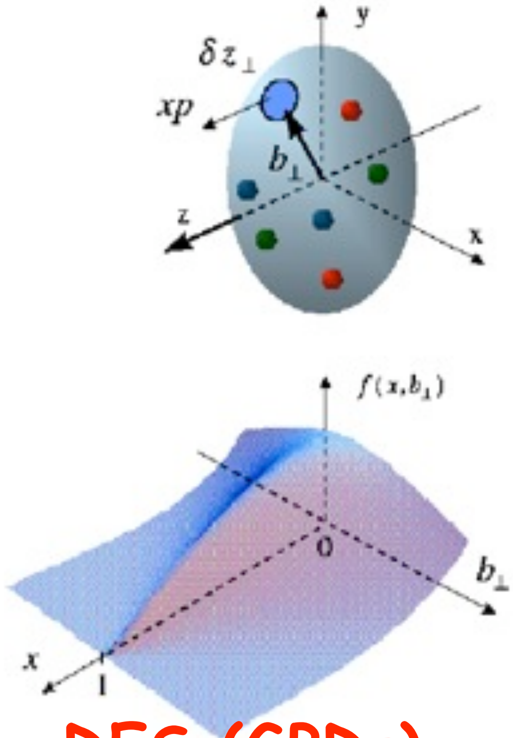
Generalized Parton Distributions : yield 3-dim quark structure of nucleon

Burkardt (2000,2003)

Belitsky,Ji,Yuan (2004)







Elastic Scattering

transverse quark distribution in coordinate space

DIS

longitudinal quark distribution in momentum space

DES (GPDs)

fully-correlated quark distribution in both coordinate and momentum space

quark orbital angular momentum

Neutron negative central charge density: inclusive-exclusive connection

Gerald A. Miller John Arrington PR C 78, 032201(R) (2008)

Goal: use GPDs to understand central negative charge density

$$H_q(x,t) = \langle p^+, \mathbf{p}', \lambda | \int \frac{dx^-}{4\pi} q_+^\dagger\left(-\frac{x^-}{2}, \mathbf{b}\right) q_+\left(\frac{x^-}{2}, \mathbf{b}\right) e^{ixp^+x^-} |p^+, \mathbf{p}, \lambda\rangle,$$

$$-t = -(p' - p)^2 = (\mathbf{p}' - \mathbf{p})^2 = Q^2.$$

$$\boxed{H_q(x,0) = q(x),\ F_1(t) = \sum_q e_q \int dx H_q(x,t).}$$

Impact parameter-dependent PDF Burkardt

Probability:

quark at $\mathbf{b}$ from cm has momentum fraction $x = k^+/p^+$

$$\rho_\perp^q(\mathbf{b},x) \equiv \langle p^+, \mathbf{R}=\mathbf{0}, \lambda| \int \frac{dx^-}{4\pi} q_+^\dagger(-\frac{x^-}{2}, \mathbf{b}) q_+(\frac{x^-}{2}, \mathbf{b}) e^{ixp^+x^-} |p^+, \mathbf{R}=\mathbf{0}, \lambda\rangle$$

$$\rho_\perp^q(\mathbf{b},x) = \int \frac{d^2q}{(2\pi)^2} e^{-i\,\mathbf{q}\cdot\mathbf{b}} H_q(x, t=-\mathbf{q}^2)$$

$$\rho(b) = \sum_q e_q \int dx\, \rho_\perp^q(\mathbf{b},x)$$

$$\mathbf{R} = 0 = \sum_i x_i \mathbf{b}_i$$

Quark of $x=1$ at $\mathbf{b}=0$

Aim: use GPDs to investigate $\rho_\perp^q(\mathbf{b}, x)$

Model GPDs- fit parton distributions and form factors

Guidal et al 05, Diehl et al 05, Ahmad et al 07, Tiburzi04

Basic idea: Drell-Yan-West relation

3 valence quarks, with power law wave function and

quark counting rules relate

DIS structure function & form factors

$\lim_{Q^2\to\infty} F_1(Q^2) = \frac{1}{Q^{2n}},\ \lim_{x\to 1} \nu W_2(x) = (1-x)^{2n-1}$

$n = 2$ relate:high x, high Q^2 , low b

$$H_v^q(x,t) = q_v(x)\exp[f_q(x)t],\ H_v^q \equiv H^q - H^{\bar{q}}.$$

Neutron ratio u/d

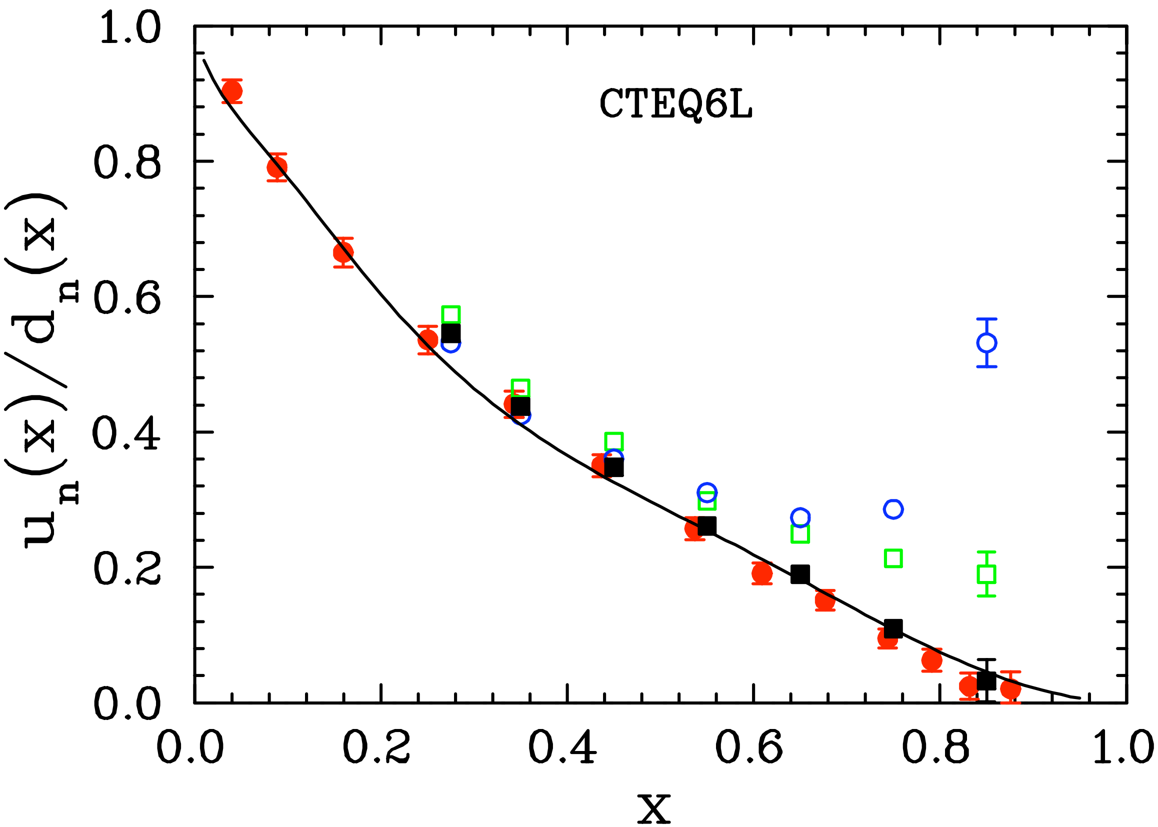
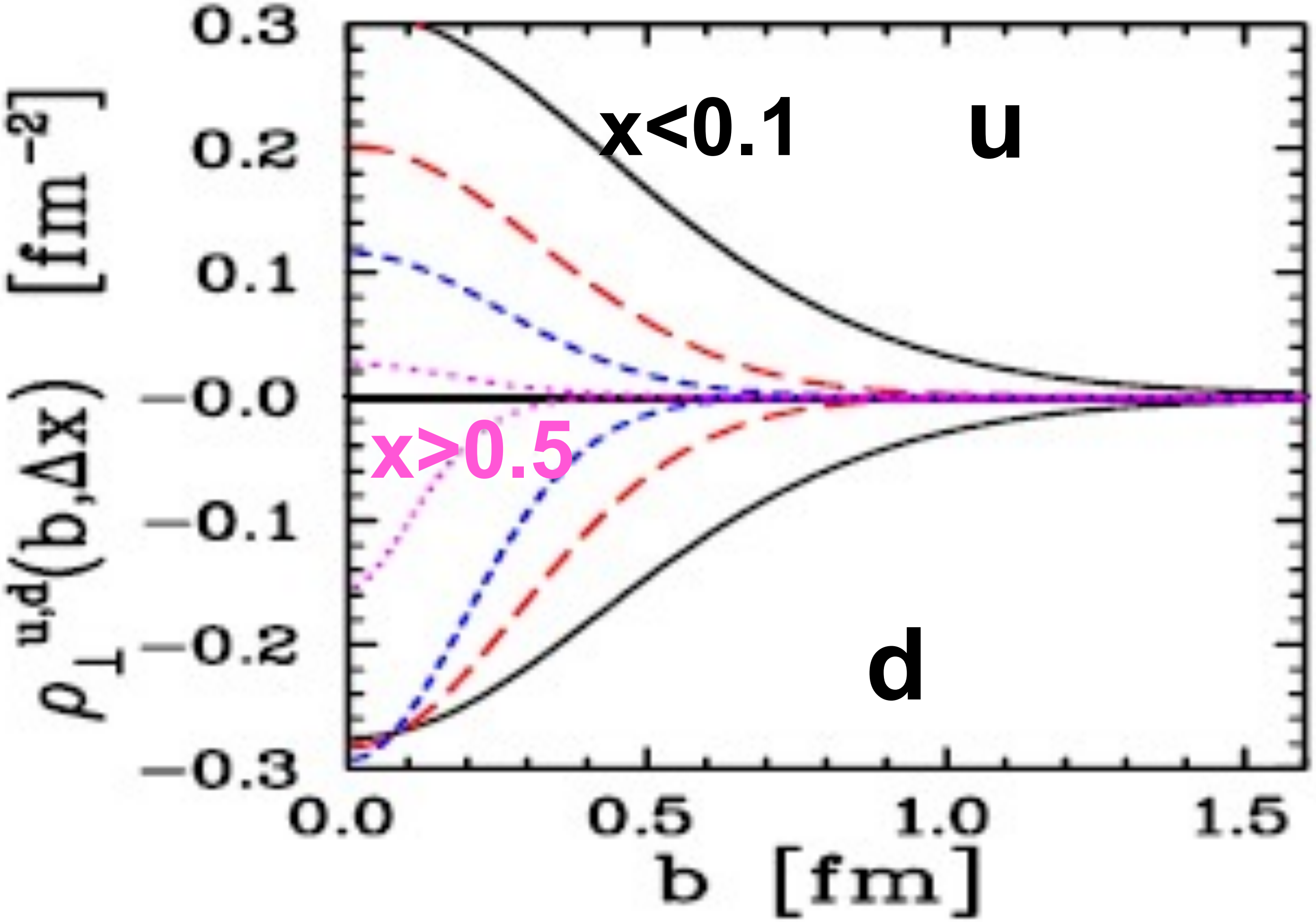


Fig. 4. Ratio of u quarks to d quarks in the neutron from several analyses of deuteron and proton data. The solid line is the CTEQ6L parameterization

Neutron d quarks dominant at high x, to be tested

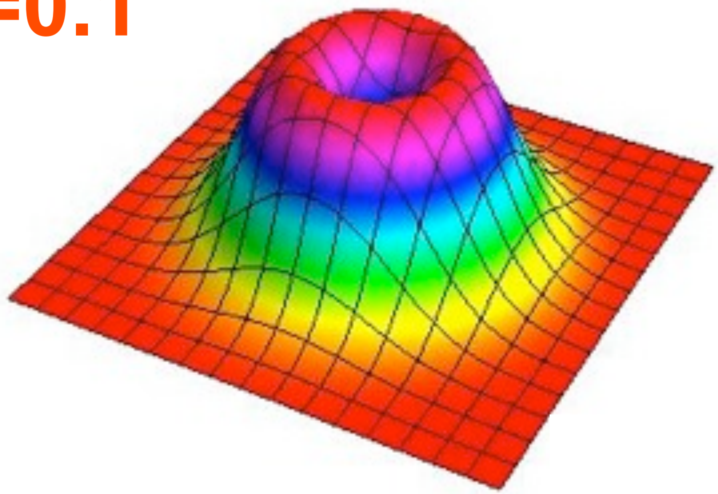
Neutron charge distribution vs x



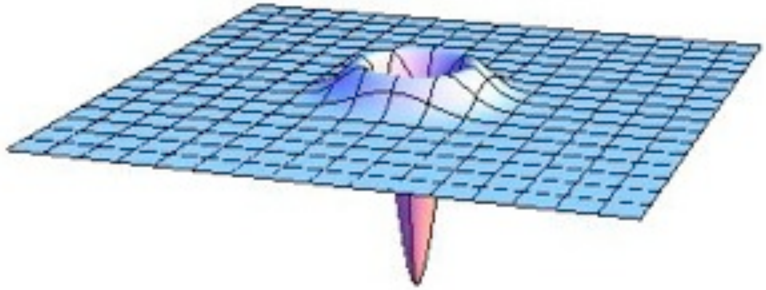
high x: d dominates at small b

Neutron ρ(b,x)

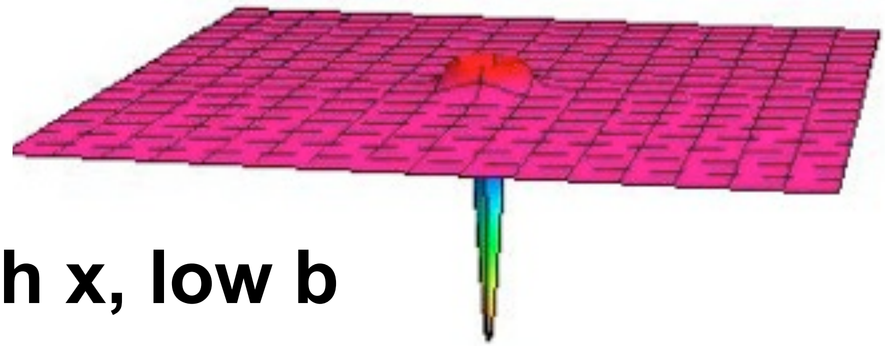
x=0.1



x=0.3

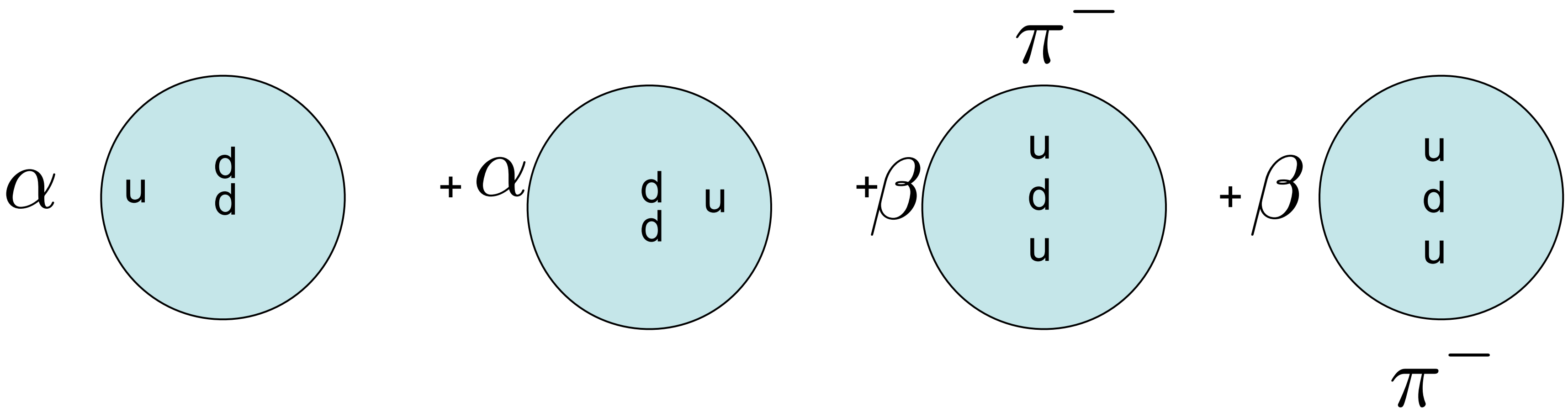


x=0.5



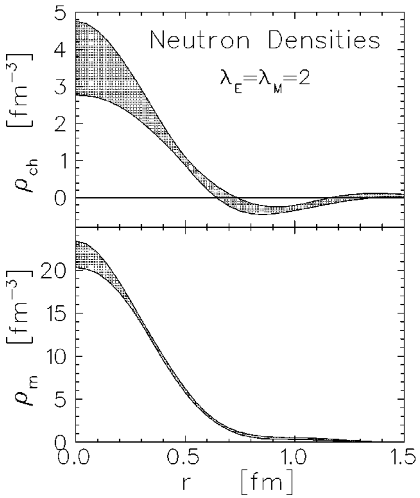
d dominates at high x, low b

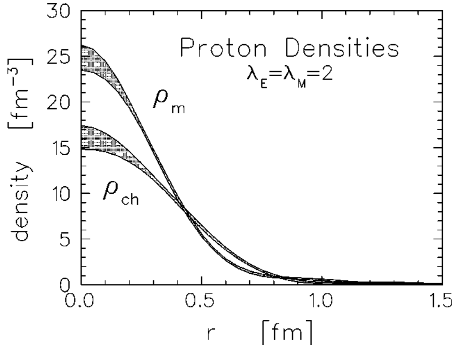
The neutron



+ .....

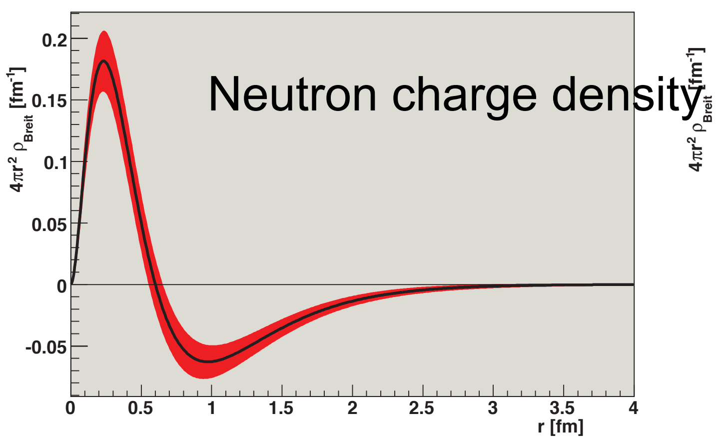
Relation between 3- dimensional and transverse densities- experimentalists love to 3 D F transform form factors

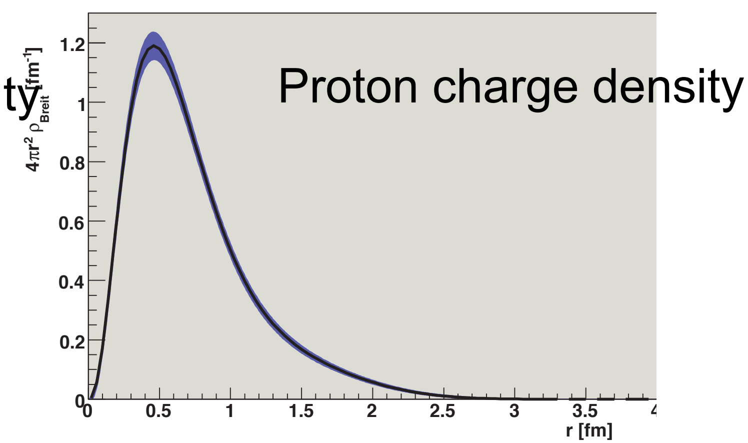




Kelly 2002

NSAC 2007





Sorry, not correct! No density interpretation of 3D FT of form factors

How to construct similar pictures to show experimental progress

$$\rho_1(r) \equiv \int \frac{d^3r}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} F_1(\vec{q}\,^2)$$

$$r = \sqrt{b^2 + z^2}$$

$$\int_{-\infty}^{\infty} dz\ \rho_1(\sqrt{b^2 + z^2}) = \int \frac{d^2b}{(2\pi^2)} e^{-i\mathbf{b}\cdot\mathbf{q}} F_1(\mathbf{q}^2) = \rho(b)$$

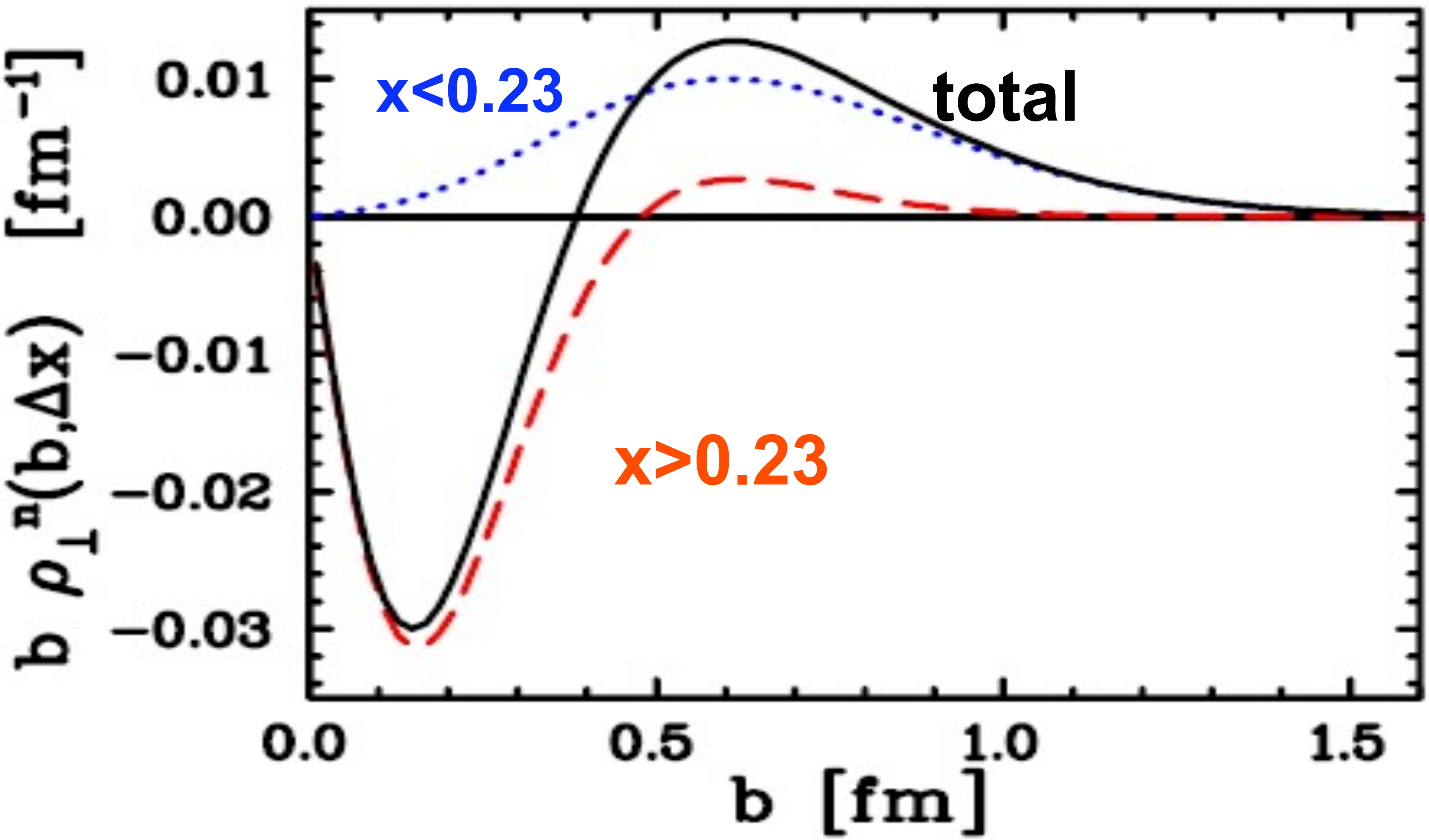
Basically-to get densities integrate the ones you had over z.

Summary

- Transverse densities give model-independent charge density in infinite momentum frame.
- 3 D FT only gives the charge density in non-relativistic, weak binding limit -e.g nuclei.
- The central transverse charge density of neutron is negative
- There are d quarks at the center of the neutron
- Transverse density can be obtained by integration over z

Spares follow

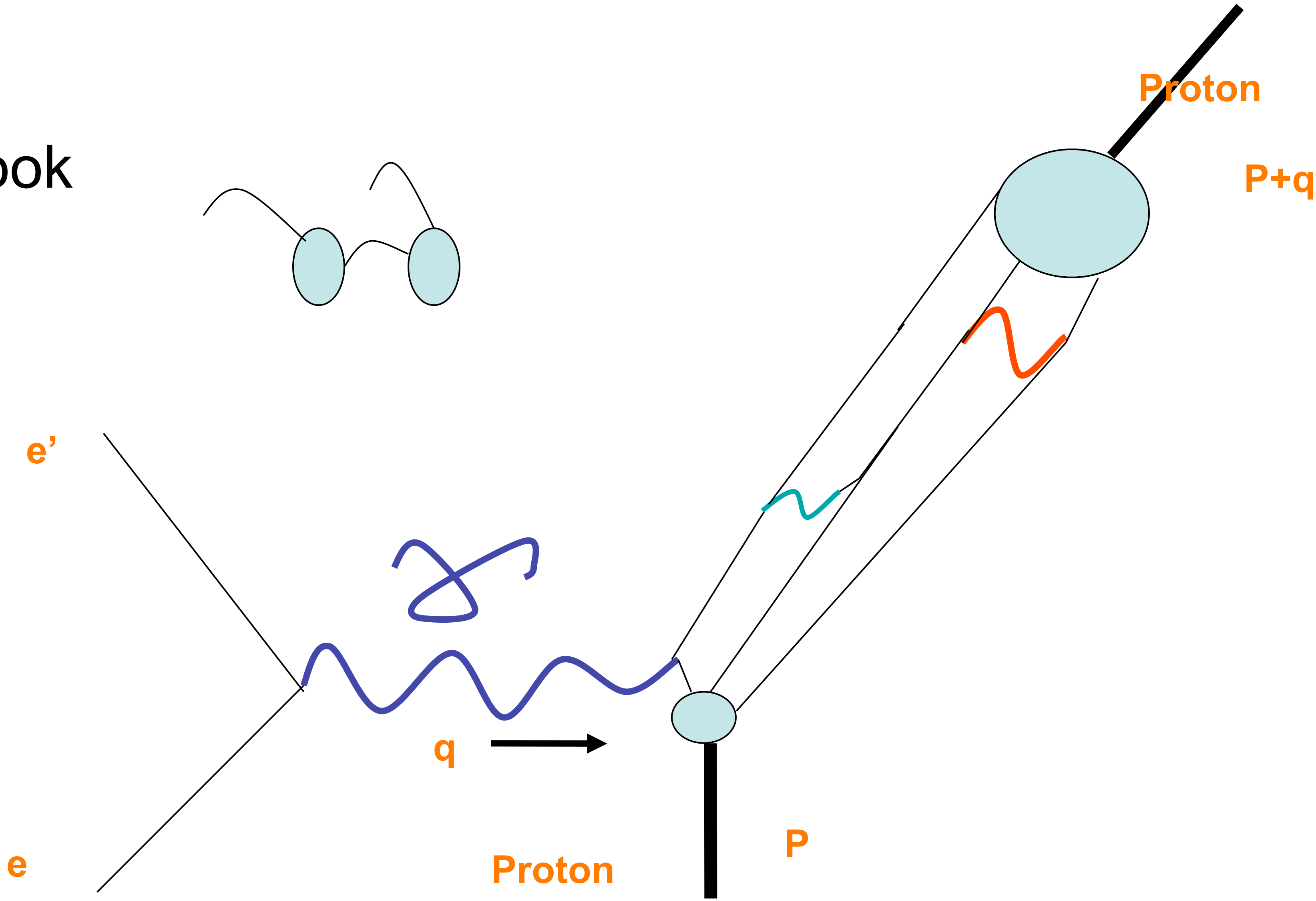
Total neutron charge distribution



d dominates at high x, low b

How to tell how big something is?

- Look



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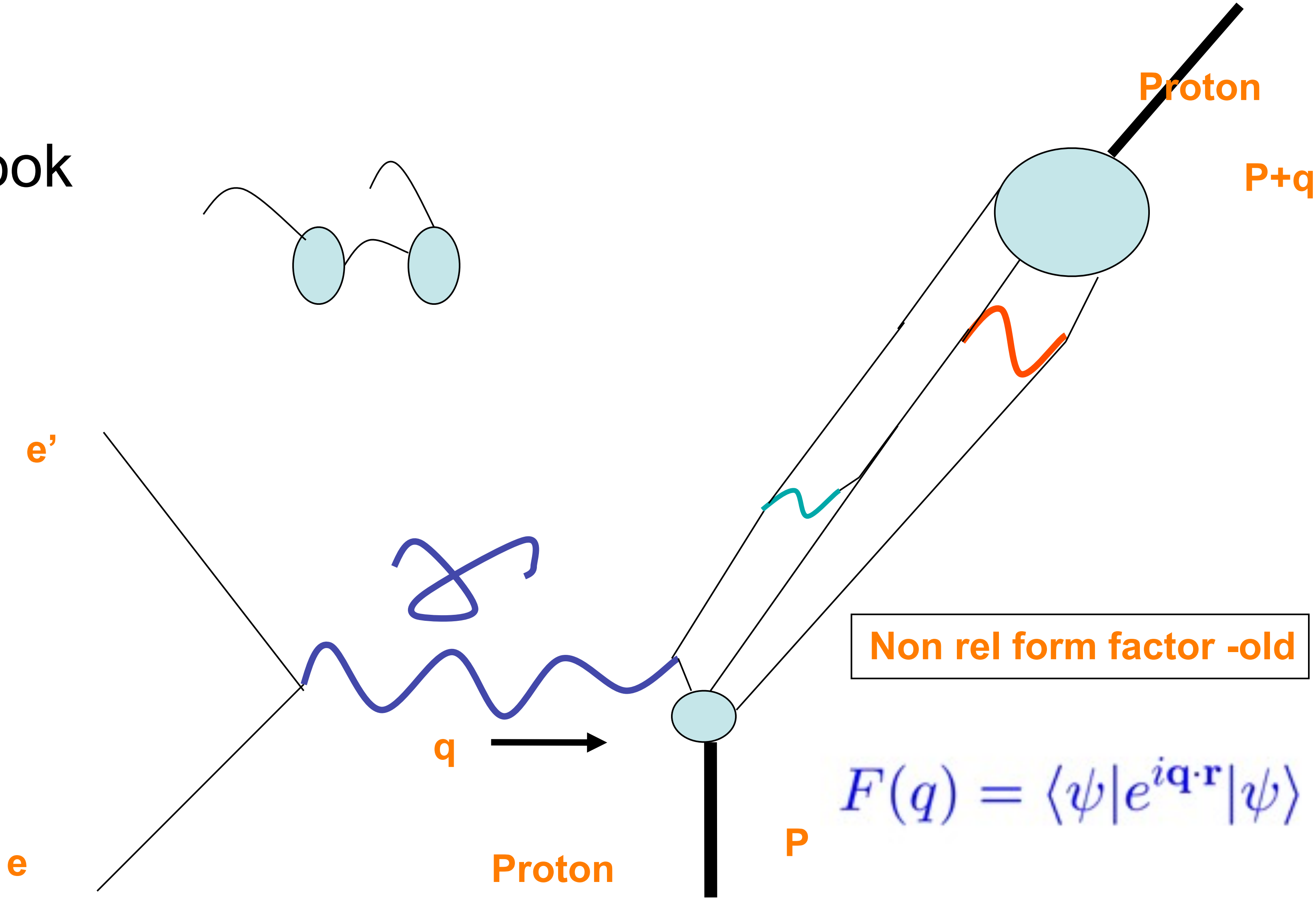
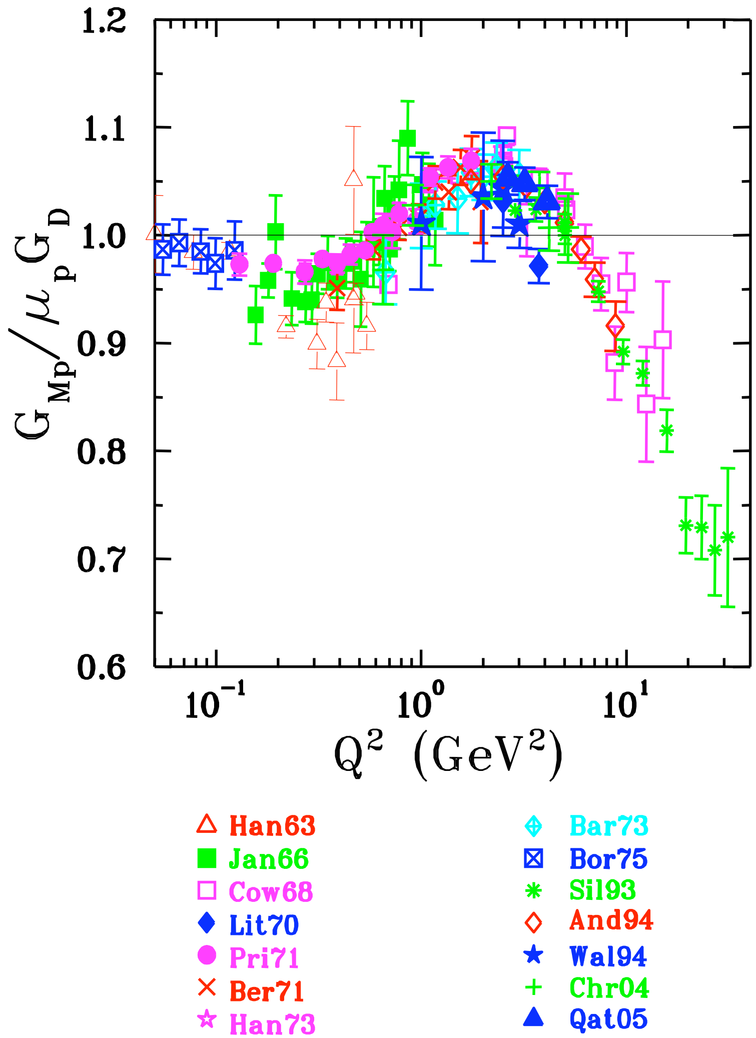


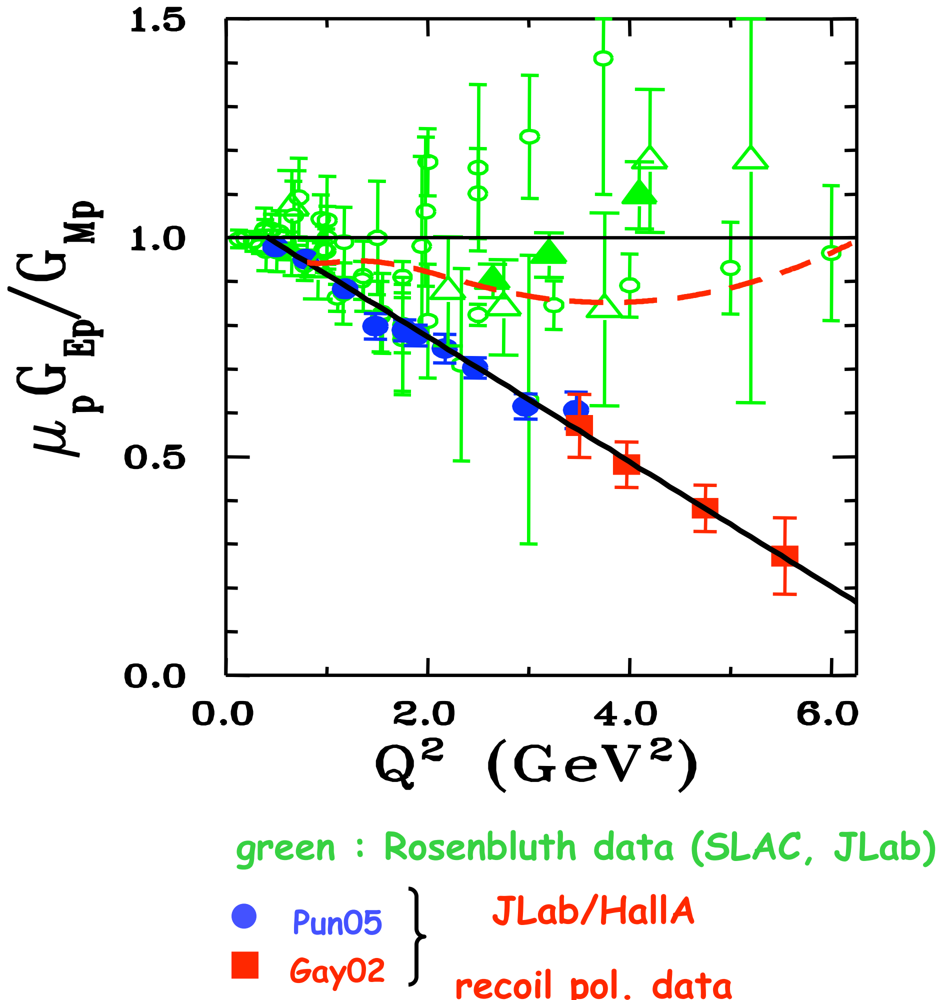
$$F(q) = \langle \psi | e^{i\mathbf{q}\cdot\mathbf{r}} | \psi \rangle$$

Structure factor

proton e.m. form factor : status

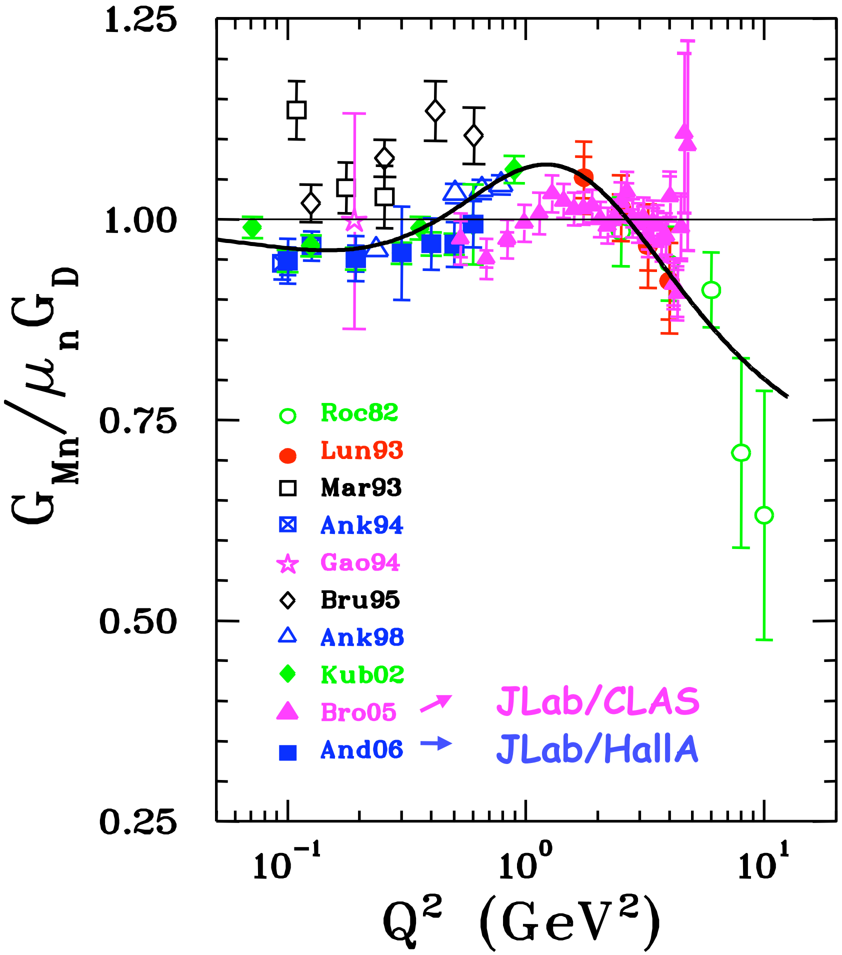


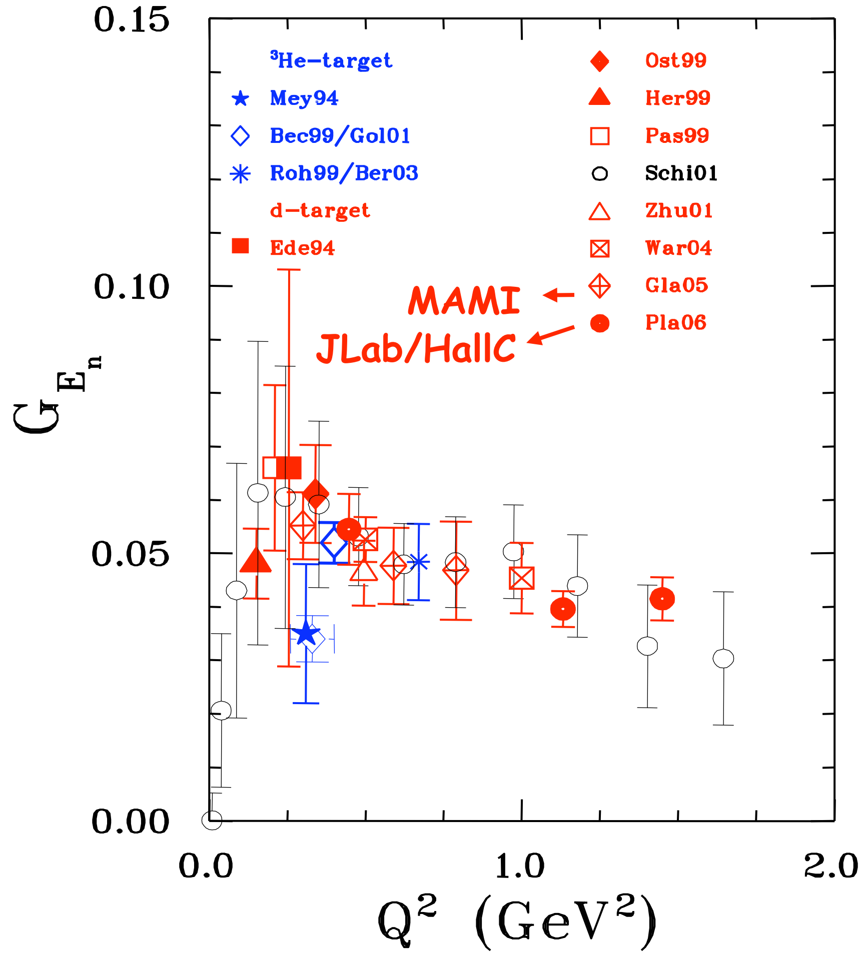
new MAMI/A1 data up to $Q^2 \approx 0.7$ GeV^2



new JLab/HallC recoil pol. exp. (spring 2008) : extension up to $Q^2 \approx 8.5$ GeV^2

neutron e.m. form factor : status





new MIT-Bates (BLAST) data for both p and n at low Q^2

new JLab/HallA double pol. exp. (spring 07) : extension up to $Q^2 \approx 3.5$ GeV^2 completed